

Noise-induced effects in high-speed reversal of magnetic dipole

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The effect of noise on the reversal of a magnetic dipole is investigated on the basis of computer simulation of the Landau-Lifshits equation. It is demonstrated that at the reversal by the pulse with sinusoidal shape, there exists the optimal duration, which minimizes the mean reversal time (MRT) and the standard deviation (jitter). Both the MRT and the jitter significantly depend on the angle between the reversal magnetic field and the anisotropy axis. At the optimal angle the MRT can be decreased by 7 times for damping $\alpha=1$ and up to 2 orders of magnitude for $\alpha=0.01$, and the jitter can be decreased from 1 to 3 orders of magnitude in comparison with the uniaxial symmetry case.

The decrease of sizes of magnetic nanoparticles used in storage media leads to the increase of fluctuations and, therefore, to increase of storage and switching errors (jitter). Thus, theoretical investigation of noise-assisted high-speed switching of magnetic dipoles is of crucial importance. The most studies are based on computer simulation of the Landau-Lifshits equation¹ describing the dynamics of the magnetic dipole in a magnetic field. It has been found that there is an optimal angle between the applied magnetic field and the anisotropy axis, which is typically around 45 degrees^{2,3}; with the decrease of the magnetic field rise time, the coercivity of magnetic particle (dynamic coercivity) increases^{4,5}. The influence of the damping and the external magnetic field amplitude on the reversal time has been investigated in^{2,6,7}; the dependence on the pulse shape of the field has been studied in⁸. All the above mentioned results have been obtained at zero temperature, i.e. without account of thermal fluctuations. However, at finite temperature the most important problem is the stable magnetic reversal: the remagnetization process must occur with minimal switching time and the standard deviation. Unfortunately, there were very little number of works, devoted to investigation of statistical characteristics of magnetic reversal processes. The mean reversal time (MRT) has been studied in Refs^{9,10}, and in Ref.¹⁰ it has been shown that during noise-assisted reversal, the noise leads to the decrease of MRT.

In the present paper the investigation of the reversal process of a magnetic dipole has been performed on the basis of computer simulation of the Landau-Lifshits equation with thermal fluctuations taken into account. It is focused on the investigation of statistical characteristics of the reversal process with the aim to find an optimal regime of reversal with the smallest mean reversal time and the standard deviation.

The dynamics of magnetic dipole is described by the Landau-Lifshits equation:

$$\frac{d\vec{M}}{dt} = -\frac{\gamma}{\beta} [\vec{M} \times \vec{H}] - \frac{\alpha\gamma}{\beta M_s} [\vec{M} \times [\vec{M} \times \vec{H}]], \quad (1)$$

where \vec{M} is the magnetization of a particle, \vec{H} is the effective magnetic field, γ is the gyromagnetic constant, $\beta = 1 + \alpha^2$, α is the damping, $M_s = |\vec{M}|$ is the saturation magnetization. The effective magnetic field contains the following components: $\vec{H} = \vec{H}_a + \vec{H}_e + \vec{H}_T$, where \vec{H}_a is the anisotropy field, \vec{H}_e is the external field, and \vec{H}_T - fluctuational field. The fluctuational field is assumed

to be white Gaussian noise with zero mean and the correlation function: $\langle H(t)_{Ti} H(t')_{Tj} \rangle = \frac{2\alpha kT}{\gamma M_s V} \delta(t - t') \delta_{ij}$, where k - Boltzmann constant, T is the temperature, and V is the volume of the magnetic particle.

Let us consider the reversal of magnetic dipole, initially magnetized along anisotropy axis and along x -axis from the state $\vec{M}[+M_s, 0, 0]$ to the state $\vec{M}[-M_s, 0, 0]$. To find the area of parameters where the fastest and the most reliable reversal occurs, as the characteristic to be studied let us choose the first passage time of a certain boundary. The mean first passage time (the mean reversal time, MRT) τ , and the standard deviation of the first passage time σ (SD, jitter) are¹¹: $\tau = \langle t \rangle = \sum_{i=1}^N t_i / N$, $\langle t^2 \rangle = \sum_{i=1}^N t_i^2 / N$, $\sigma = \sqrt{\langle t^2 \rangle - \langle t \rangle^2}$, where t_i is the first passage time of an absorbing boundary and $N \geq 10000$ is the number of realizations. As in Ref.⁶ let us choose the boundary as the passage of the point $\vec{M}[0, M_y, M_z]$.

In the calculations it is convenient to use the parameters, related to the magnetic recording media³: $\alpha=0.1$, $\gamma=1.76 \cdot 10^7 \text{ Hz/Oe}$, $M_s=360 \text{ Oe}$, $V=2 \cdot 10^3 \text{ nm}^3$, the anisotropy constant $K=7.2 \cdot 10^5 \text{ erg/cm}^3$. The static coercivity is $H_c=2K/M_s=4000 \text{ Oe}$. For modeling we take the amplitude of the magnetic field to be $H_0=6000 \text{ Oe}$. It is known that the driving by the signal with sharp fronts leads to the minimal MST¹². However, the pulses used in real recording media systems have finite rise time³. As an example of a driving with smooth fronts we consider the sinusoidal pulse $\vec{H}_e = \vec{e} H_0 \sin \pi t / t_p$ with the width t_p (see the inset of Fig. 1), where \vec{e} is unitary vector of magnetic field direction. If the switching during t_p does not happen, the computation is continued for $\vec{H}_e=0$ until some long period of time t_f , much larger than any other relaxation time scale. Our aim is to find the parameter range, for which the reversal by a smooth pulse takes place quickly and reliably.

The Landau-Lifshits equation with noise has been computed both by the Heun method programmed in Fortran and by the specialized package SIMMAG (Simulation of MicroMAGnets), developed in the laboratory of mathematical modeling of IPM RAS.

It is known that for zero temperature $T = 0$ the reversal of the dipole by the longitudinal field, $\theta = 0$, does not occur, since the dipole is in the equilibrium state, even if this state is unstable. The presence of thermal fluctuations allows to move the dipole away from this equilibrium state. In Fig. 1 the plots of the mean reversal time τ and the standard deviation σ are presented. First of all, it is seen that both τ and σ have minima as

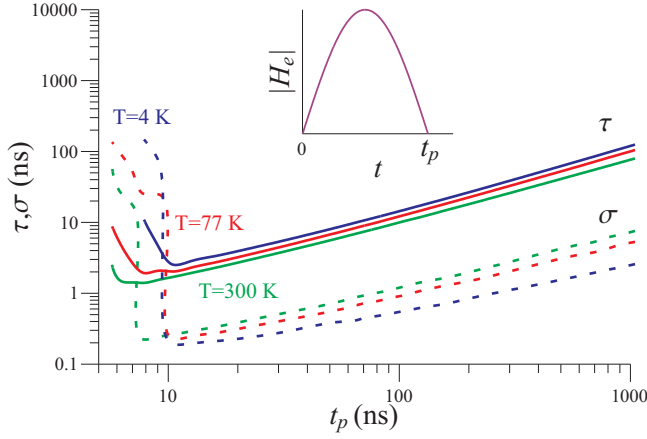


FIG. 1: The mean reversal time (solid curves) and the standard deviation (dashed curves) versus pulse width for the zero angle between anisotropy axis and the driving field $\theta=0$. Inset: the driving pulse.

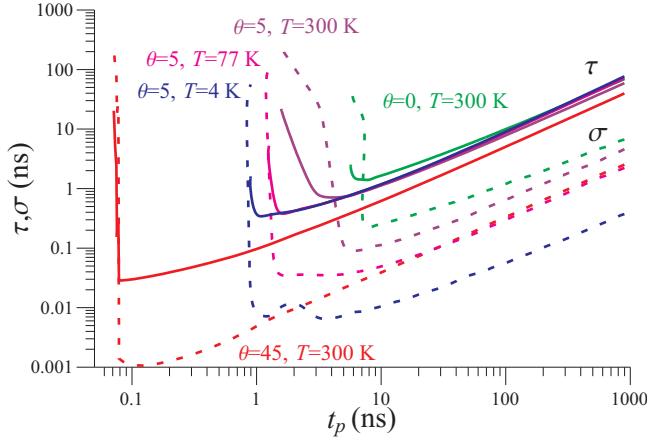


FIG. 2: The MRT (solid curves) and SD (dashed curves) versus pulse width for different angles between anisotropy axis and external field and different temperatures for $\theta = 5^\circ$.

functions of the driving pulse width. This indicates that both these temporal characteristics can be minimized by the optimal choice of pulse duration. Similar effect has recently been observed for Josephson junctions¹². The decrease of the MRT at large durations is due to the fact that with decrease of the width the potential barrier disappears faster. With further shortening of the pulse, the magnetization does not have enough time for the complete reversal during t_p , so the MRT increases. This, actually, means that for rather short pulses the transition occurs due to effect of fluctuations (the so-called noise-induced switching). The standard deviation with decrease of the pulse width behaves similarly to τ , but the curves for different temperatures cross each other. For long pulses smaller temperature leads to smaller σ , but for short pulses - vice versa. This is also explained by the transition from the regime of switching by the external field to the noise-induced escapes, since it is well-known, see, e.g.,¹¹, that in the noise-induced regime the SD is approximately equal to the MRT.

Let us point out few more peculiarities, which are clearly visible in Fig. 1: higher temperature leads to

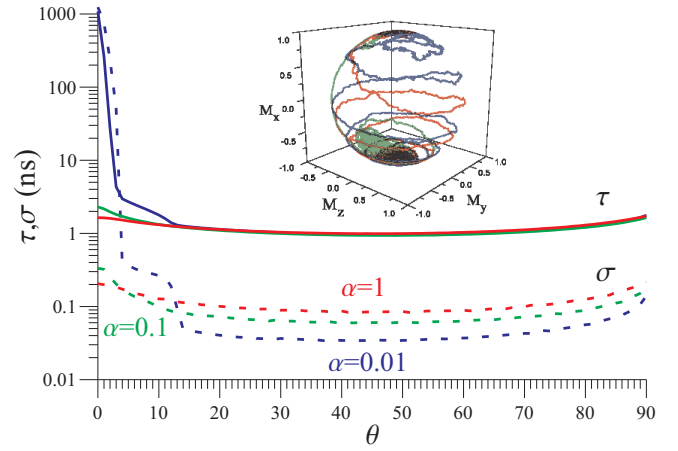


FIG. 3: The MRT (solid curves) and SD (dashed curves) versus the angle between anisotropy axis and external magnetic field for different values of damping α , $t_p = 8$ ns and temperature $T = 300$ K. Inset: trajectories of the magnetization for different angles, $\alpha = 0.1$, $t_p = 15.7$ ns, $T = 300$ K; $\theta = 0^\circ$ - blue curve, $\theta = 5^\circ$ - red curve, $\theta = 45^\circ$ - green curve.

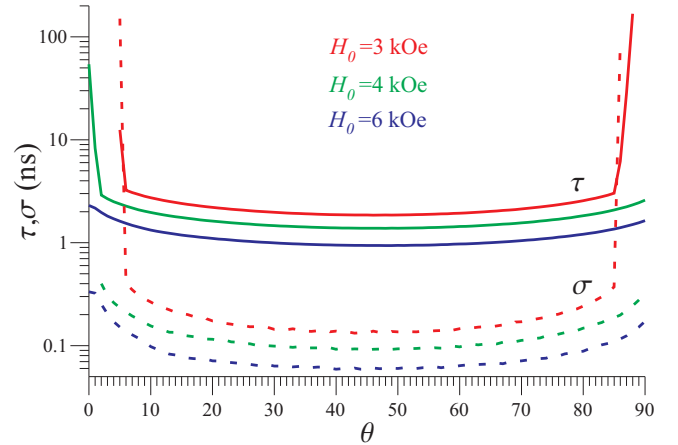


FIG. 4: The MRT (solid curves) and SD (dashed curves) versus the angle between anisotropy axis and external magnetic field for different values of magnetic field amplitude, $t_p = 8$ ns and temperature $T = 300$ K.

smaller MRT, i.e. noise allows to speed up the reversal, which agrees with the predictions of Ref.¹⁰, but contradicts with the results of Ref.¹². Besides, the dependence $\sigma \sim \sqrt{T}$ usual for Josephson junctions at large pulse widths, is not also reproduced here, from Fig. 1 one can see that $\sigma(T)$ dependence is slower than \sqrt{T} . The explanation of such an unusual behavior is the specific nature of the reversal process at $\theta = 0$: in difference with the Josephson junctions¹², due to location of initial condition at the unstable equilibrium point, the reversal is impossible at zero temperature and the deterministic reversal trajectory does not exist. That is why fluctuations help to leave the unstable initial state and namely this leads to the described above peculiarities. It should be noted that the above investigated case $\theta = 0$, which is mostly studied analytically¹³⁻¹⁵, is degenerate: first, technically the angle between the anisotropy axis and the external field can be set up with a certain precision; second, the MRT is largest in this case, see below.

In Fig. 2 the MRT and SD are given for three different values of angles $\theta=0^\circ, 5^\circ, 45^\circ$ for $T=300$ K and three different temperatures for $\theta=5^\circ$. First, let us focus on the curves for $\theta=5^\circ$. In spite that at large t_p one can see little decrease of MRT with increase of the temperature, at small t_p around minimum the opposite effect of noise delayed switching is clearly visible, quite similar to the one observed before for Josephson junctions¹², and in general case of nonlinear systems¹⁶. Besides, here the SD behaves as $\sigma \sim \sqrt{T}$, see Ref.¹².

From Fig. 2 it is obvious that for $\theta=45^\circ$ the reversal is faster and more stable than for $\theta=0^\circ, 5^\circ$ at all other equal conditions. Besides, the difference between MRT for the angles $\theta=0^\circ$ and $\theta=5^\circ$ is two times, while the difference of SD is about three times. For the cases $\theta=5^\circ$ and $\theta=45^\circ$ the gain is even larger, more than one order for MRT and almost two orders for SD. This means that the reversal process principally depends on the precession of the magnetic dipole, and can not be described by a simple two-state model. This result gives the quantitative substantiation for the idea to use the tilted magnetic field to speed up the reversal process¹⁷, and also to use additional weak perpendicular magnetic field for the same purpose¹⁸, which actually leads to the tilt of the aggregate magnetic field. To roughly estimate the probability of nonswitching of a dipole by one pulse with the duration t_p , the probability density of switching times can be considered as Gaussian with the mean τ and SD σ . Then the probability of nonswitching is: $p=\frac{1}{2}\text{erfc}((t_p - \tau)/\sqrt{2}\sigma)$. For $T=300$ K at the minimum of σ we get $p=10^{-48}$ even for $\theta=0^\circ$. However, for $\theta=0^\circ, 5^\circ$ at the minimum of τ we get unacceptably large probabilities 0.001 ($p=10^{-56}$ for $\theta=45^\circ$).

In Fig. 3 the MRT and SD are presented versus the angle between anisotropy axis and external magnetic field θ for different values of damping α , $t_p=8$ ns and $T=300$ K. For $\theta \rightarrow 0$, smaller values of damping lead to larger values of both τ and σ as it must (large values of τ and σ for $\theta \rightarrow 0$ and $\alpha=0.01$ mean that in this range of parameters the noise-induced reversal occurs). For larger angles, however, the MRT nearly coincide, while

the SD is smaller for smaller values of α . To understand why it is so, let us plot the trajectories of the magnetization for $\theta=0^\circ, 5^\circ, 45^\circ$, see the inset of Fig. 3 for $\alpha=0.1$, $t_p=15.7$ ns. One can see that the precession is largest for $\theta=0^\circ$, for $\theta=5^\circ$ the number of turns is smaller, and for $\theta=45^\circ$ the crossing of the boundary $\vec{M}[0, M_y, M_z]$ occurs even without precession. This explains why the reversal in the latter case happens much faster than for $\theta=0^\circ$ and has little dependence on α in the limit $\alpha \ll 1$, see Fig. 3. Since the length of the path is nearly the same for different α (the MRT nearly coincide) and the noise intensity is proportional to the damping, this obviously leads to smaller SD for smaller α .

In Fig. 4 the MRT and SD versus angle θ are presented for different values of the external magnetic field amplitude. It is seen that starting from values $20^\circ - 30^\circ$ and up to $\sim 70^\circ$ there are flat minima of MRT and SD that corresponds to the range of angles where the fastest and the most reliable reversals are realized. For the temperature $T = 300$ K the reversal occurs even up to the amplitude value $H_0 = 3$ kOe, which is two time smaller than the static coercive field H_c .

In the present paper the effect of noise on the reversal of a magnetic dipole has been investigated on the basis of computer simulation of the Landau-Lifshits equation. It has been demonstrated that at the reversal by the pulse with sinusoidal shape, there exists the optimal duration, which minimizes the mean reversal time (MRT) and the standard deviation (SD, jitter). Also, both the MRT and the jitter significantly depend on the angle between the reversal magnetic field and the anisotropy axis. At the optimal angle the MRT can be decreased by 7 times for $\alpha=1$ and up to 2 orders of magnitude for $\alpha=0.01$; the jitter can be decreased from 1 to 3 orders of magnitude (for α from 1 to 0.01) in comparison with the uniaxial symmetry case. For optimal angles the SD decreases with decrease of the damping. It has been demonstrated that fluctuations can not only decrease the reversal time, as it has been known before for the magnetic systems and is correct for small angles only, but it can also significantly increase the reversal time.

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